12-4 Operations with Events

Ex4. You and your best friend both have old cars. The probability of your car working is .92 and the probability of your friend's car working is .68. What is the probability that:

$.92 \cdot .68 = .6256$

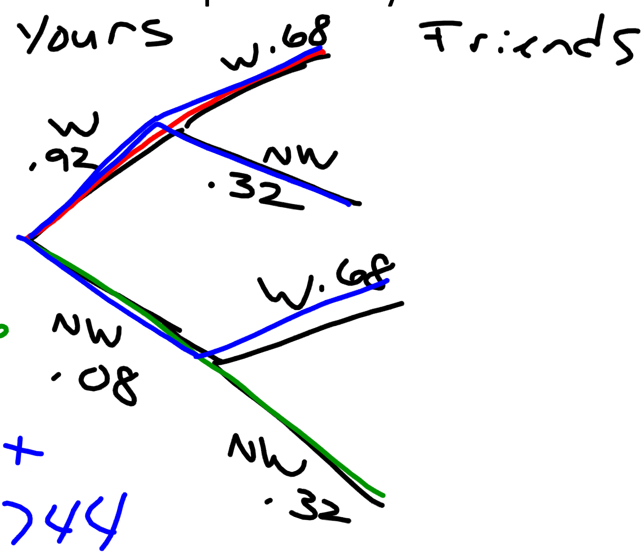
a.) Both cars are working

b.) Neither car is working

$.08 \cdot .32 = .0256$

c.) At least 1 working

$.92 \cdot .68 + .92 \cdot .32 + .08 \cdot .68 = .9744$



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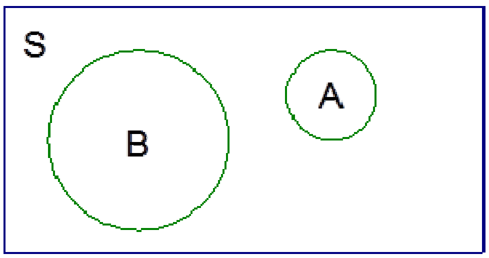
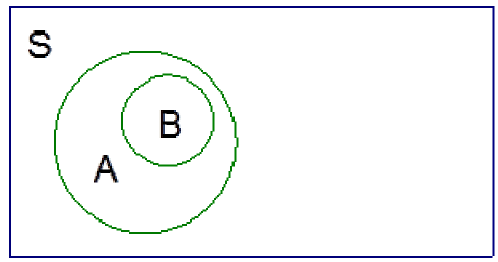
Conditional Probability

If finding out that one events has already occurred changes the probability that a second event will occur, the 2 events are said to be dependent.

For example – the calc class problem earlier.



Let A and B be mutually exclusive events. If we find out B has occurred, we know A cannot occur so we should change P(A)=0.



Let B be a subset of A. If we find out B has occurred, we know A must also have occurred so we should change P(A)=1.

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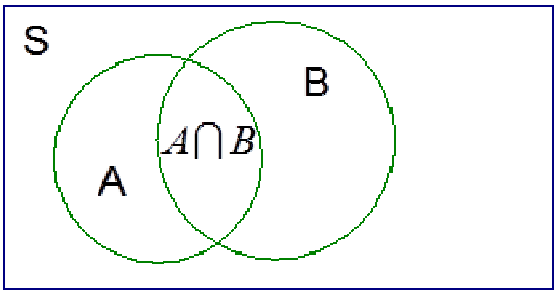
The probability we assign to an event can change if we know that some other event has occurred. This idea is key to understanding conditional probability.

P(A) = the probability that A occurs

P(A/B) = the probability that A occurs given that B already occurred.



If we learn B has occurred, we can restrict attention to just those outcomes in B and disregard the rest of S, so we have a new sample space that is just B.

For A to have occurred in addition to B, the conditional probability of A given B is:

$$P(A / B) = \frac{P(A \cap B)}{P(B)}$$

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Ex5. A bag contains 1 yellow marble, 6 red marbles, and 5 blue marbles. Find:

a.) P(Y) = $\frac{1}{12}$ P(R) = $\frac{1}{2}$ P(B) = $\frac{5}{12}$

You draw one marble from the bag and without returning it, you draw a second marble. Find.

b.) P(B/Y)= 5/11 P(B/R)= 5/11 P(B/B)= 4/11

P(R/Y)= 6/11 P(R/R)= 5/11 P(R/B)= 6/11

P(Y/Y)= 0 P(Y/R)= 1/11 P(Y/B)= 1/11

Independence

Two events are independent if learning that one occurred does not affect the probability that the other occurred. That is, if P(A/B) = P(A) and vice versa.

Two events are independent if and only if $P(A \cap B) = P(A) \cdot P(B)$. Otherwise the events are dependent.

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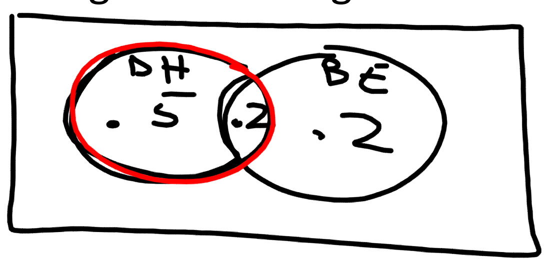
Ex6. The probability that a girl has dark hair is .7, the probability that she has blue eyes is .4, and the probability that she has both dark hair and blue eyes is .2.

B = Blue Eyes   D = Dark Hair

a.) Draw a Venn Diagram showing this information.



b.) Find the probability that a girl has neither dark hair nor blue eyes.

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= .7 + .4 - .2 = .9$

$1 - P(A \cup B)$

$1 - .9 = .1$

c.) Given that a girl has dark hair, find the probability that she also has blue eyes.

$$P(B/D) = \frac{P(B \cap D)}{P(D)} = \frac{.2}{.7} = \frac{2}{7}$$

$$\approx .286$$

d.) Given that a girl does not have dark hair, find the probability that she has blue eyes.

$$P(B/D') = \frac{.2}{.3} = \frac{2}{3} \approx .667$$

e.) Are the characteristics of having dark hair and having blue independent? Explain your answer.

Method #1

Is $P(D) = P(D/B)$

$P(B) = P(B/D)$

$.7 \stackrel{?}{=} \frac{.2}{.4}$

$.7 \neq \frac{1}{2}$

$.4 \stackrel{?}{=} \frac{.2}{.7}$

$.4 \neq .286$

Not independent

Dependent

Method #2

Is $P(B \cap D) = P(B) \cdot P(D)$

$.2 \stackrel{?}{=} .4 \cdot .7$

$.2 \neq .28$

Not independent

Dependent

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